

HIP-1998-82/TH
KANAZAWA-98-20

November 1998

Running of Soft Parameters in Extra Space-Time Dimensions

TATSUO KOBAYASHI ^{(1),†}, JISUKE KUBO ⁽²⁾,
MYRIAM MONDRAGÓN ^{(3),*} and GEORGE ZOUPANOS ^{(4),**}

⁽¹⁾ *Department of Physics, High Energy Physics Division, University of Helsinki
and*

Helsinki Institute of Physics, FIN-00014 Helsinki, Finland

⁽²⁾ *Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan*

⁽³⁾ *Instituto de Física, UNAM, Apdo. Postal 20-364, México 01000 D.F., México*

⁽⁴⁾ *Physics Department, Nat. Technical University, GR-157 80 Zografou, Athens, Greece.*

Abstract

The evolution of the parameters including those in the soft supersymmetry-breaking (SSB) sector is studied in the minimal supersymmetric standard model (MSSM) with a certain set of Kaluza-Klein towers which has been recently considered by Dienes *et al.* We use the continuous Wilson renormalization group technique to derive the matching condition between the effective, renormalizable and original, unrenormalizable theories. We investigate whether the assumption on a large compactification radius in the model is consistent with the gauge coupling unification, the $b-\tau$ unification and the radiative breaking of the electroweak gauge symmetry with the universal SSB terms. We calculate the superpartner spectrum under the assumption of the universal SSB parameters to find differences between the model and the MSSM.

[†]Partially supported by the Academy of Finland (no. 37599).

^{*}Partially supported by the UNAM Papiit project IN110296.

^{**}Partially supported by the E.C. projects, FMBI-CT96-1212 and ERBFMRXCT960090, and the Greek projects, PENED95/1170; 1981.

1 Introduction

Recently, motivated by the works of refs. [1, 2] in which the strong coupling limit of heterotic superstrings has been considered, there have been renewed interests to consider Kaluza-Klein theories with a large compactification radius [3]–[32]. In an extreme case, the radius could be in the range of submillimeter, whereas the Kaluza-Klein excitations with masses $\lesssim 10$ TeV will become observable in future collider experiments [4]–[10]. These scenarios could be even embedded into various superstring models with an anisotropic compactification [3]–[25], especially into models based on Type I superstring which should describe the strong coupling limit of the $SO(32)$ heterotic superstring [2].

It is clear that the qualitative nature of the traditional unification scenario changes [16, 26, 27] if the fields of the standard model (SM) or of the minimal supersymmetric standard model (MSSM) feel the existence of extra dimensions whose scale is significantly smaller than the ordinary GUT scale $M_{\text{GUT}} \sim 10^{15-16}$ GeV. We would like to emphasize that, strictly speaking, the unification in those scenarios takes place not in 4 rather in $D = 4 + \delta$ dimensions in which the original theory is formulated. On one hand, quantities near and above M_{GUT} , including M_{GUT} itself, are $D = 4 + \delta$ dimensional quantities. At energies much below the compactification scale, on the other hand, all the massive Kaluza-Klein states decouple so that we have a four dimensional effective theory. Therefore, there must be a certain matching condition between the four dimensional effective and $D = 4 + \delta$ dimensional theories. Clearly, the four dimensional theory does not know anything about the matching condition, but they can be derived from the original $D = 4 + \delta$ dimensional theory. In fact, in the treatment of Dienes *et al.* [16] on the massive Kaluza-Klein excitations, one needs an outside information about the infrared and ultraviolet cut off to define a finite, four dimensional, low-energy effective theory. As we will see in sect. 2, we will derive the matching condition, in contrast to ref. [16], from the requirement that the evolution equations of couplings in the effective theory smoothly go over in the large compactification-radius limit to what one finds in uncompactified, original, $D = 4 + \delta$ dimensional theory. Specifically, we will employ the continuous Wilson renormalization group (RG) approach [33] which can be formulated in any space-time dimensions. Of various existing formulations

[34, 35] of the continuous Wilson RG in literature, we will follow the formulation of ref. [34]. It turns out that the small discrepancy in the matching condition compared with that of ref. [16] has no significant effect in the application to the model of ref. [16] which we also will consider in this paper.

Given the matching condition, we will be staying in four dimensions to extend the method of ref. [16] so as to include the soft supersymmetry-breaking (SSB) sector. To this end, we will make use of the recent progress on the renormalization properties of SSB parameters [36]–[39]. It has been shown that the ultraviolet divergences of the SSB parameters are simply related to those of the corresponding supersymmetric parameters [36] so that the β -functions of the SSB parameters can be easily obtained by applying certain differential operators on the anomalous dimensions and the gauge coupling β -function of the supersymmetric theory [38, 39]. This method works only for four dimensional theories when using a mass-independent renormalization scheme such as the dimensional reduction scheme [36, 38, 39]. So it is not obvious that the method can be applied straightforwardly to softly broken supersymmetric theories with Kaluza-Klein towers, because one defines the theory by cut off. Fortunately, the (ultraviolet) divergent parts in one-loop order are independent of renormalization scheme and have a simple structure so that the massive Kaluza-Klein excitations do not disturb the above mentioned relations among the divergent parts of the SSB parameters and those of the supersymmetric parameters. In the second-half of sect 2, we will consider the model of ref. [16], the MSSM with a certain set of Kaluza-Klein towers, and calculate the one-loop β -functions for the SSB parameters above the compactification scale $\mu_0 = R^{-1}$.

Given the β -functions, we can discuss various aspects of the model in a more concrete fashion, which will be the subject of sect. 3. We first consider the gauge coupling unification and find that the smaller the μ_0 is, the larger is the predicted value of the QCD coupling $\alpha_3(M_Z)$, in accord with the result of ref. [27]. We however need more detailed information on a possible theory above μ_0 to control the corrections such as the threshold effect at M_{GUT} in order to give a more precise prediction of $\alpha_3(M_Z)$. Here we will consider the possibility that the level of $U(1)_Y$ can be different from the usual $SU(5)$ -motivated value $5/3$.

We will calculate the mass of the bottom quark under the assumption of the $b - \tau$ Yukawa coupling unification for the given top quark mass. Our analysis points out that

the $b - \tau$ unification can be consistent with a large compactification radius. In the final part of sect. 3, we investigate RG effects on the SSB parameters, where we assume that they are universal at the GUT scale. In particular, we study the possibility of achieving the radiative electroweak symmetry breaking, the nature of the lightest superparticle (LSP), and the constraint coming from the negative (mass)² of the stau. As we will conclude in sect. 4, the basic low-energy ($\lesssim O(1)$ TeV) feature of the MSSM remains unchanged even if μ_0 is as small as $\sim O(10)$ TeV, but there exist a certain chance in the superpartner spectrum to experimentally discriminate the model from the MSSM.

2 β -functions in extra dimensions

2.1 Large radius limit and matching condition

Suppose that we would like to study low-energy physics of a Kaluza-Klein theory which is defined in $D = 4 + \delta$ dimensions with extra δ dimensions compactified. The theory is not renormalizable, and presumably trivial, but it can be well defined by introducing an ultraviolet cut off Λ_0 . The natural framework to study the low-energy physics is provided by the continuous Wilson renormalization group (RG) [33], which can be realized in terms of an integro-differential equation [34, 35]. Here we would like to follow the formulation of ref. [34]. Since we expect that, in the limit that the radius of the compactified dimensions approaches infinity, the result goes over to what one finds in the uncompactified case, we briefly sketch below the treatment of ref. [34] in the case of a scalar theory in uncompactified Euclidean D dimensions.

The basic idea to the non-perturbative RG approach [34]¹ is to divide the field $\phi(p)$ in the momentum space into low and high energy modes,

$$\phi(p) = \theta(\Lambda - |p|)\phi_{<}(p) + \theta(|p| - \Lambda)\phi_{>}(p) , \quad (1)$$

and integrate out the high energy modes in the path integral to define the effective theory:

$$S_{\text{eff}}[\phi_{<}, \Lambda] = -\ln \left\{ \int \mathcal{D}\phi_{>} e^{-S[\phi_{>}, \phi_{<}]} \right\} , \quad (2)$$

¹See also ref. [40]. The continuous Wilson renormalization RG approach is called sometimes the non-perturbative RG approach.

where S_{eff} is the Wilson effective action. It turns out that the difference

$$\delta S_{\text{eff}} = S_{\text{eff}}[\phi_<, \Lambda + \delta\Lambda] - S_{\text{eff}}[\phi_<, \Lambda] \quad (3)$$

for an infinitesimal $\delta\Lambda$ becomes a Gaussian path integral which can be in fact carried out. That is, it is possible to calculate $\Lambda(\partial S_{\text{eff}}/\partial\Lambda)$ to write down a formal expression for the RG flow equation of the effective theory in the form

$$\Lambda \frac{\partial S_{\text{eff}}}{\partial\Lambda} = \mathcal{O}(S_{\text{eff}}) , \quad (4)$$

where \mathcal{O} is a non-linear operator acting on the functional S_{eff} . The explicit expression for \mathcal{O} was first obtained by Wegner and Houghton [34], but in practice, the RG equation (4) cannot be solved exactly. There are various approaches to find approximate solutions to the Wegner-Houghton (W-H) equation, and one of the successful ones is the so-called local potential approximation [41]–[43], which we would like to adopt here. In this approximation, one makes an ansatz for the solution to the W-H equation (4) in the form

$$S_{\text{eff}} = \int d^D x \left(\frac{1}{2} \partial_M \phi \partial_M \phi + V(\phi^2) \right) , \quad (5)$$

and finds that the W-H equation (4) reduces to a partial differential equation for the potential V [42, 43],

$$\Lambda \frac{\partial V}{\partial\Lambda} = -\frac{A_D}{2} \ln(1 + V' + 2\rho V'') - DV - (2 - D)\rho V' , \quad (6)$$

where we have defined:

$$\rho = \frac{\phi^2}{2} , \quad V' = \frac{dV}{d\rho} , \quad A_D = \frac{2^{1-D}}{\pi^{D/2}\Gamma(D/2)} , \quad (7)$$

and all the quantities are made dimensionless by multiplying them with an appropriate power of Λ . If we furthermore assume that the potential V is a power series in ρ , i.e.,

$$V(\rho) = \sum_{n=0} \frac{\tilde{\lambda}_n(\Lambda)}{n!} \rho^n , \quad (8)$$

we find [43]

$$\Lambda \frac{d\lambda_0}{d\Lambda} = -\frac{A_D}{2} \ln(1 + \Lambda^{-2}\lambda_1) \Lambda^D , \quad (9)$$

$$\Lambda \frac{d\lambda_1}{d\Lambda} = -A_D \frac{3\lambda_2/2}{(1 + \Lambda^{-2}\lambda_1)} \Lambda^{D-2} , \quad (10)$$

$$\Lambda \frac{d\lambda_2}{d\Lambda} = -A_D \left(-\frac{9\lambda_2^2/2}{(1 + \Lambda^{-2}\lambda_1)^2} \Lambda^{D-4} + \frac{5\lambda_3/2}{(1 + \Lambda^{-2}\lambda_1)} \Lambda^{D-2} \right) , \quad (11)$$

$$\Lambda \frac{d\lambda_3}{d\Lambda} = -A_D \left(\frac{27\lambda_2^3}{(1 + \Lambda^{-2}\lambda_1)^3} \Lambda^{D-6} - \frac{45\lambda_2\lambda_3/2}{(1 + \Lambda^{-2}\lambda_1)^2} \Lambda^{D-2} + O(\lambda_4) \right) , \quad (12)$$

$$\vdots \quad (13)$$

where we have defined the dimensionful couplings λ_n as

$$\lambda_n = \Lambda^{D(1-n)+2n} \tilde{\lambda}_n . \quad (14)$$

The set of the evolution equations above systematically includes the effects of higher dimension operators, and should be approximately valid for $\Lambda \gg 1/R$ if some of the spatial dimensions are compactified. In deriving the set of evolution equations (9)-(12), we have neglected the effect of the compactification, but it is clear from the discussion above that we could in principle introduce this effect into the non-perturbative RG framework. We leave this program to future work.

Recently, Dienes *et al.* [16] have suggested a method to study low-energy physics of a Kaluza-Klein theory, and their method is formulated within a framework of $D = 4$ dimensional, renormalizable theory. Next we would like to see whether their result in the $R \rightarrow \infty$ limit goes over to what one finds in the non-perturbative RG approach. To this end, we assume that we can neglect $\lambda_1, \lambda_3, \dots$ in the evolution of λ_2 –the scalar quartic coupling. Then eq. (11) can be written as

$$\Lambda \frac{d\lambda_2}{d\Lambda} = A_D \frac{9}{2} \lambda_2^2 \Lambda^{D-4} . \quad (15)$$

Now assuming that $\delta = D - 4$ dimensions are compactified on a circle of a fixed radius R , we find from eq. (15) that the evolution equation of the dimensionless quartic coupling $\lambda = (2\pi R)^\delta \lambda_2$ of the four dimensional theory becomes

$$\Lambda \frac{d\lambda}{d\Lambda} = \frac{1}{8\pi^2} \frac{9}{2(1 + \delta/2)} X_\delta \lambda^2 (R\Lambda)^\delta , \quad X_\delta = \frac{\pi^{\delta/2}}{\Gamma(1 + \delta/2)} , \quad (16)$$

where X_δ (the volume of a δ dimensional sphere of radius one) has been introduced in ref. [16]. We will compare this result with the one which we obtain by using the method of ref.

[16]. As we will see, they differ from each other. To clarify the origin of the discrepancy, we first would like to follow the method of ref. [16] for the present scalar theory, and derive the evolution equation of λ . We will then give an argument why two results are different and motivate how to obtain the agreement.

As in ref. [16] we assume that $\delta = D - 4$ dimensions are compactified on a circle of a fixed radius R , where x and y stand for the 4 and δ dimensional coordinates, respectively. The scalar field satisfying the periodic boundary condition

$$\phi(x, y) = \phi(x, y + 2\pi R) \quad (17)$$

can be expanded as

$$\phi(x, y) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \cdots \sum_{n_\delta=-\infty}^{\infty} \phi_n(x) \exp(in \cdot y/R), \quad (18)$$

where $n = (n_1, n_2, \dots, n_\delta)$ with $n_i \in \mathbf{Z}$, and $n \cdot y = \sum_{i=1}^{\delta} n_i y_i$. The starting Lagrangian is

$$\mathcal{L}_D = \frac{1}{2}(\partial_M \phi \partial_M \phi + m_0^2 \phi^2) + \frac{\lambda}{8} \phi^4, \quad M = 1, \dots, 4, 5, \dots, 4 + \delta. \quad (19)$$

To define the four dimensional theory, we rescale the field and the coupling λ as

$$\phi_n(x) \rightarrow (2\pi R)^{-\delta/2} \phi_n(x), \quad \lambda \rightarrow (2\pi R)^\delta \lambda. \quad (20)$$

The (four dimensional) mass squared of the Kaluza-Klein modes $\phi_n(x)$ is given by

$$m_n^2 = m_0^2 + \frac{n \cdot n}{R^2}, \quad (21)$$

where m_0 is the mass of the zero mode ϕ_0 , which we would like to neglect as has been done in ref. [16]. For energies above $\mu_0 = R^{-1}$, the Kaluza-Klein excitations are observable, and we may expect that in the $R \rightarrow \infty$ limit the theory behaves as a full $4 + \delta$ dimensional theory.

Following ref. [16], we compute the one-loop correction $\Pi_D^{(4)}$ to the four point vertex function with zero external momenta to obtain the one-loop correction to the coupling λ . We find

$$\Pi_D^{(4)} = \frac{9}{2} \lambda^2 \int \frac{d^4 p}{(2\pi)^4} \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \cdots \sum_{n_\delta=-\infty}^{\infty} \frac{1}{(p^2 + m_n^2)^2} \quad (22)$$

$$= \frac{\lambda^2}{8\pi^2} \frac{9}{2} \int_0^\infty \frac{dt}{t} \left(\frac{1}{2} \right) [\vartheta_3(it/\pi R^2)]^\delta, \quad (23)$$

where ϑ_3 is one of the Jacobi theta functions

$$\vartheta_3(\tau) = \sum_{n=-\infty}^{\infty} \exp(i\pi n^2) , \quad (24)$$

and we have used

$$\frac{1}{A^2} = \int_0^{\infty} dt \exp(-At) , \quad \int \frac{d^4 p}{(2\pi)^4} \exp(-tp^2) = \frac{1}{16\pi^2 t^2} . \quad (25)$$

Note that the t integral is ultraviolet as well as infrared divergent ($\text{Dim}[t] = -2$). Dienes *et al.* [16] introduced an ultraviolet and infrared cut off to define the t integral:

$$\int_0^{\infty} dt \rightarrow \int_{r\Lambda^{-2}}^{r\mu_0^{-2}} dt , \quad r = \pi(X_\delta)^{-2/\delta} , \quad (26)$$

where X_δ is defined in (16), and $\mu_0 = 1/R$. We emphasize that the factor r cannot be obtained within the framework of the four dimensional theory, and so the explicit expression given in (26) comes from an outside information, to which we will come later. Assuming that $t/R^2 \ll 1$ so that $\vartheta(it/\pi R^2)$ may be approximated as $R\sqrt{\pi/t}$, we perform the t integration to obtain

$$\Pi_D^{(4)} = \frac{\lambda^2}{8\pi^2} \frac{9X_\delta}{2\delta} \left(\left(\frac{\Lambda}{\mu_0} \right)^\delta - 1 \right) . \quad (27)$$

Then we compute the β -function for λ :

$$\Lambda \frac{d\lambda}{d\Lambda} = \frac{\lambda^2}{8\pi^2} \frac{9X_\delta}{2} \left(\frac{\Lambda}{\mu_0} \right)^\delta , \quad \lambda^{-1}(\Lambda) = \lambda^{-1}(\Lambda_0) - (\Pi_D^{(4)}(\Lambda) - \Pi_D^{(4)}(\Lambda_0)) . \quad (28)$$

Comparing this result with the evolution equation (16), we now see that they differ by a factor $(1 + \delta/2)$.

This difference may be understood in the following way. In the treatment of ref. [16], one has to define the infrared and ultraviolet cut off, i.e., the factor r appearing in the t integral in (26). They fix r by interpreting that the correction to the β -function (coming from the massive excitations) is proportional to the number of Kaluza-Klein excitations with masses smaller than Λ . This number is approximately proportional to the volume of δ dimensional sphere of radius ΛR , that is, $X_\delta(\Lambda/\mu_0)^\delta$. This interpretation does not lead to a β -function that in the large R limit approaches the corresponding β -function of the full D dimensional

theory as we have seen above. In the full theory, we have a D dimensional integral in the momentum space in the form

$$\lim_{\Lambda \rightarrow \infty} \Lambda \frac{\partial}{\partial \Lambda} \int \frac{d^D q}{(2\pi)^D} K(q^2) = A_D \lim_{\Lambda \rightarrow \infty} \Lambda \frac{\partial}{\partial \Lambda} \int^\Lambda d|q| |q|^{D-1} K(|q|^2) , \quad (29)$$

where A_D (the D dimensional angular integral) is defined in (7)², whereas the interpretation of Dienes *et al.* [16] would correspond to the expression

$$\begin{aligned} \lim_{\Lambda \rightarrow \infty} \int \frac{d^\delta k}{(2\pi)^\delta} \Lambda \frac{\partial}{\partial \Lambda} \int \frac{d^4 p}{(2\pi)^4} K(p^2) &= A_\delta A_4 \lim_{\Lambda \rightarrow \infty} \int^\Lambda d|k| |k|^{\delta-1} \\ &\times \Lambda \frac{\partial}{\partial \Lambda} \int^\Lambda d|p| |p|^3 K(|p|^2) . \end{aligned} \quad (30)$$

Assuming that the function $K(x)$ has the form $K(x) = x^{2N}$ where N is some arbitrary number, we find that the difference of the two integrals above is exactly the factor $(1 + \delta/2)$, the same factor that appears between the β -functions (16) and (28). This means that if one would multiply r defined in (26) with the factor $(1 + \delta/2)^{2/\delta}$, one would get β -functions that in the large radius limit go over to those obtained in the non-perturbative RG approach. So we would like to suggest to rescale the cut off factor r in the t integration (26) as

$$r \rightarrow (1 + \delta/2)^{2/\delta} r , \quad (31)$$

or equivalently to replace X_δ according to

$$X_\delta \rightarrow Y_\delta = \frac{\pi^{\delta/2}}{\Gamma(2 + \delta/2)} , \quad (32)$$

where X_δ is defined in eq. (16).

2.2 Extension to the soft supersymmetry-breaking sector

We now discuss the running of the SSB parameters. To this end, we follow the notation of ref. [38] and consider first a generic $N = 1$ supersymmetric gauge theory with the superpotential

$$W(\Phi) = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j , \quad (33)$$

² This is the reason that the all β -functions in the full theory are proportional to A_D (see (15)). Besides, there are only one-loop corrections in the non-perturbative RG approach we are adopting [34, 42, 43].

and with the SSB part L_{SSB} given by [36]

$$L_{SSB}(\Phi, W) = - \left(\int d^2\theta \eta \left(\frac{1}{6} h^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} b^{ij} \Phi_i \Phi_j + \frac{1}{2} M W_A^\alpha W_{A\alpha} \right) + \text{h.c.} \right) - \int d^4\theta \tilde{\eta} \eta \overline{\Phi^j} (m^2)_j^i (e^{2gV})_i^k \Phi_k , \quad (34)$$

where $\eta = \theta^2$, $\tilde{\eta} = \tilde{\theta}^2$ are the external spurion superfields and $\theta, \tilde{\theta}$ are the usual Grassmann parameters, and M is the gaugino mass. The β -functions of the M, h and m^2 parameters can be computed from [38, 39]:

$$\beta_M = 2\mathcal{O} \left(\frac{\beta_g}{g} \right) , \quad (35)$$

$$\beta_b^{ij} = \gamma^i_l b^{lj} + \gamma^j_l b^{il} - 2\gamma_{1l}^i \mu^{lj} - 2\gamma_{1l}^j \mu^{il} , \quad (36)$$

$$\beta_h^{ijk} = \gamma^i_l h^{ljk} + \gamma^j_l h^{ilk} + \gamma^k_l h^{ijl} - 2\gamma_{1l}^i Y^{ljk} - 2\gamma_{1l}^j Y^{ilk} - 2\gamma_{1l}^k Y^{ijl} , \quad (37)$$

$$(\beta_{m^2})^i_j = \left[\Delta + X \frac{\partial}{\partial g} \right] \gamma^i_j , \quad (38)$$

$$\mathcal{O} = \left(M g^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial Y^{lmn}} \right) , \quad (39)$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2 \frac{\partial}{\partial g^2} + \tilde{Y}_{lmn} \frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn} \frac{\partial}{\partial Y^{lmn}} , \quad (40)$$

where $(\gamma_1)^i_j = \mathcal{O}\gamma^i_j$, $Y_{lmn} = (Y^{lmn})^*$, and $\tilde{Y}^{ijk} = (m^2)_l^i Y^{ljk} + (m^2)_l^j Y^{ilk} + (m^2)_l^k Y^{ijl}$.

The formulae for the β -functions of the SSB parameters (35)–(38) have been derived from the observation that in a class of renormalization schemes the divergent parts of the SSB parameters are simply related to those in the symmetric theory, Y^{ijk} and μ^{ij} . It is, however, not exactly known in which class of renormalization schemes these formulae have their validity. In fact, the quantity $X \sim O(g^3)$ in eq. (38) is explicitly computed only in two-loop order [44] and depends on the renormalization scheme employed ³.

Since however we are interested only in the one-loop approximation to the β -functions, the problem mentioned above is irrelevant because the divergent parts in one-loop order are independent of renormalization scheme. Moreover, as we can see from the calculation of the contribution coming from the massive Kaluza-Klein excitations to the β -functions, these excitations do not disturb the relations among the divergent parts of the SSB parameters

³ There exists an indirect method (which is based on a RG invariance argument) to fix the exact form of X [45, 46] in the Novikov-Shifman-Vainshtein-Zakharov [47] renormalization scheme.

and Y^{ijk} and μ^{ij} at least in one-loop order. Therefore, we can easily compute the one-loop β -functions of the SSB parameters above μ_0 by applying the eqs. (35)–(38) on the β -functions and anomalous dimensions which contain the one-loop contributions coming from the massive Kaluza-Klein excitations.

To be more specific we consider the model of ref. [16], the MSSM with a certain Kaluza-Klein towers. As ref. [16] we assume that only the gauge boson and Higgs supermultiplets of the MSSM have the towers of Kaluza-Klein states and that the lepton and quark supermultiplets are stuck at a fixed point of an orbifold on which the δ dimensional internal space is compactified so that they have no towers of Kaluza-Klein states. Under these assumptions, the one-loop β -functions of the gauge couplings and the one-loop anomalous dimensions above and below μ_0 become [16]:

$$(16\pi^2)\beta_1 = \begin{cases} g_1^3 (6 + \frac{6}{5}(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta) \\ \frac{33}{5} g_1^3 \end{cases} \quad (41)$$

$$(16\pi^2)\beta_2 = \begin{cases} g_2^3 (4 - 6(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta) \\ g_2^3 \end{cases}, \quad (42)$$

$$(16\pi^2)\beta_3 = \begin{cases} g_3^3 (3 - 12(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta) \\ -3 g_3^3 \end{cases}, \quad (43)$$

$$(16\pi^2)\gamma_{t_L} = \begin{cases} Y_\delta (\frac{\Lambda}{\mu_0})^\delta (g_t^2 + g_b^2 - (\frac{1}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2)) \\ g_t^2 + g_b^2 - (\frac{1}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2) \end{cases}, \quad (44)$$

$$(16\pi^2)\gamma_{t_R} = \begin{cases} Y_\delta (\frac{\Lambda}{\mu_0})^\delta (2g_t^2 - (\frac{8}{15}g_1^2 + \frac{8}{3}g_3^2)) \\ 2g_t^2 - (\frac{8}{15}g_1^2 + \frac{8}{3}g_3^2) \end{cases}, \quad (45)$$

$$(16\pi^2)\gamma_{b_R} = \begin{cases} Y_\delta (\frac{\Lambda}{\mu_0})^\delta (2g_b^2 - (\frac{2}{15}g_1^2 + \frac{8}{3}g_3^2)) \\ 2g_b^2 - (\frac{2}{15}g_1^2 + \frac{8}{3}g_3^2) \end{cases}, \quad (46)$$

$$(16\pi^2)\gamma_{\tau_L} = \begin{cases} Y_\delta (\frac{\Lambda}{\mu_0})^\delta (g_\tau^2 - (\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2)) \\ g_\tau^2 - (\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2) \end{cases}, \quad (47)$$

$$(16\pi^2)\gamma_{\tau_R} = \begin{cases} Y_\delta (\frac{\Lambda}{\mu_0})^\delta (2g_\tau^2 - \frac{6}{5}g_1^2) \\ 2g_\tau^2 - \frac{6}{5}g_1^2 \end{cases}, \quad (48)$$

$$(16\pi^2)\gamma_{H_u} = \begin{cases} 3g_t^2 - (\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2) \\ 3g_t^2 - (\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2) \end{cases}, \quad (49)$$

$$(16\pi^2)\gamma_{H_d} = \begin{cases} 3g_b^2 + g_\tau^2 - (\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2) \\ 3g_b^2 + g_\tau^2 - (\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2) \end{cases}, \quad (50)$$

where $g_{t,b,\tau}$ are the Yukawa couplings for the top, bottom and tau, respectively, we have neglected the Yukawa couplings of the first and second generations. (Y_δ is defined in (32).) Here we have used the fact [16] that in the model of [16], the contributions of the excited Kaluza-Klein states to the anomalous dimensions of the matter supermultiplets have the same form as the massless mode contribution, and that those of the Higgs supermultiplets due to $N = 2$ supersymmetry in the excited sector vanish.

The one-loop β -functions for the Yukawa couplings can be computed from

$$\beta_{ijk} = g_{ijk} (\gamma_i + \gamma_j + \gamma_k), \quad (51)$$

and we find that for energies above μ_0

$$(16\pi^2)\beta_t = g_t [3g_t^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 + (Y_\delta/2) (\frac{\Lambda}{\mu_0})^\delta (6g_t^2 + 2g_b^2 - \frac{17}{15}g_1^2 - 3g_2^2 - \frac{32}{3}g_3^2)], \quad (52)$$

$$(16\pi^2)\beta_b = g_b [3g_b^2 + g_\tau^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 + (Y_\delta/2) (\frac{\Lambda}{\mu_0})^\delta (2g_t^2 + 6g_b^2 - \frac{1}{3}g_1^2 - 3g_2^2 - \frac{32}{3}g_3^2)], \quad (53)$$

$$(16\pi^2)\beta_\tau = g_\tau [3g_b^2 + g_\tau^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 + (Y_\delta/2) (\frac{\Lambda}{\mu_0})^\delta (6g_\tau^2 - 3g_1^2 - 3g_2^2)]. \quad (54)$$

The gaugino mass β -functions are:

$$(16\pi^2)\beta_{M_1} = M_1 g_1^2 (6 + \frac{6}{5}(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta), \quad (55)$$

$$(16\pi^2)\beta_{M_2} = M_2 g_2^2 (4 - 6(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta), \quad (56)$$

$$(16\pi^2)\beta_{M_3} = M_3 g_3^2 (3 - 12(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta), \quad (57)$$

where we have used eq. (35). One of the consequences of (35) is that in one-loop order the

relation

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} \quad (58)$$

holds above as well as below μ_0 . The β -functions of the trilinear couplings above μ_0 are:

$$\begin{aligned} (16\pi^2)\beta_{h_t} = & h_t [9g_t^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2] + g_t [\frac{3}{5}g_1^2M_1 + 3g_2^2M_2] \\ & + (Y_\delta/2) \left(\frac{\Lambda}{\mu_0} \right)^\delta \left(h_t [18g_t^2 + 2g_b^2 - \frac{17}{15}g_1^2 - 3g_2^2 - \frac{32}{3}g_3^2] \right. \\ & \left. + g_t [4g_bh_b + \frac{34}{15}g_1^2M_1 + 6g_2^2M_2 + \frac{64}{3}g_3^2M_3] \right) , \end{aligned} \quad (59)$$

$$\begin{aligned} (16\pi^2)\beta_{h_b} = & h_b [9g_b^2 + g_\tau^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2] + g_b [2g_\tau h_\tau + \frac{3}{5}g_1^2M_1 + 3g_2^2M_2] \\ & + (Y_\delta/2) \left(\frac{\Lambda}{\mu_0} \right)^\delta \left(h_b [18g_b^2 + 2g_t^2 - \frac{1}{3}g_1^2 - 3g_2^2 - \frac{32}{3}g_3^2] \right. \\ & \left. + g_b [4g_th_t + \frac{2}{3}g_1^2M_1 + 6g_2^2M_2 + \frac{64}{3}g_3^2M_3] \right) , \end{aligned} \quad (60)$$

$$\begin{aligned} (16\pi^2)\beta_{h_\tau} = & h_\tau [3g_b^2 + 3g_\tau^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2] + g_\tau [6g_bh_b + \frac{3}{5}g_1^2M_1 + 3g_2^2M_2] \\ & + (Y_\delta/2) \left(\frac{\Lambda}{\mu_0} \right)^\delta \left(h_\tau [18g_\tau^2 - 3g_1^2 - 3g_2^2] \right. \\ & \left. + g_\tau [6g_1^2M_1 + 6g_2^2M_2] \right) , \end{aligned} \quad (61)$$

where we have used eqs. (35)–(38).

The β -functions for μ_H and B are the same as those below μ_0 , because the anomalous dimensions for the Higgs superfields $\gamma_{H_u}, \gamma_{H_d}$ are the same. From the same reason, the β -functions for $m_{H_u}^2, m_{H_d}^2$ are the same as those below μ_0 . To obtain the β -functions for the soft squared masses of the leptons and quarks above μ_0 , we simply have to multiply their β -functions below μ_0 with the factor

$$Y_\delta \left(\frac{\Lambda}{\mu_0} \right)^\delta , \quad (62)$$

where Y_δ is defined in (32).

3 Predictions from unification

In the previous section we have derived the matching condition and extended the method of ref. [16] to include the SSB sector. In this section we would like to apply this result

to a model which has been considered in ref. [16]. This model is nothing but the MSSM with the Kaluza-Klein towers which are present only in the gauge supermultiplets and Higgs supermultiplets ⁴, and we have given the one-loop RG functions of this model in the previous section. We however will restrict ourselves to the case with $\delta = 1$, because the main feature of our results will not drastically change for $\delta > 1$. To simplify the situation, we assume through out our analyses a uniform SUSY threshold M_S and that $M_S = 1$ TeV. Unless we notice explicitly, we study the evolution of the dimensionless parameters such as gauge couplings below $\mu_0 = R^{-1}$ at the two-loop level, along with the experimental values,

$$M_\tau = 1.777 \text{ GeV}, \quad M_Z = 91.188 \text{ GeV}, \quad (63)$$

$$\alpha_{\text{EM}}^{-1}(M_Z) = 127.9 + \frac{8}{9\pi} \log \frac{M_t}{M_Z}, \quad (64)$$

$$\sin^2 \theta_W(M_Z) = 0.2319 - 3.03 \times 10^{-5} T - 8.4 \times 10^{-8} T^2, \quad (65)$$

where $T = M_t/[\text{GeV}] - 165$. Here M_τ and M_t are the physical tau and top quark masses, where we take $M_t = 174.1$ GeV in our analyses ⁵. (See ref. [49] for more details of the method of our analyses.) The evolution of all the parameters above $\mu_0 = R^{-1}$ as well as the evolution of the SSB parameters for the whole range of the energy scale will be studied at the one-loop level.

In the following discussions we change our notation for the GUT scale: $M_{\text{GUT}} \rightarrow M_X$ while we use M_Y for M_{GUT} when considering the level of $U(1)_Y$ as free.

3.1 Gauge coupling unification

We begin by considering the unification of the gauge couplings of the model. Fig. 1 shows a representative example of the running of the gauge couplings $\alpha_a = g_a^2/4\pi$ ($a = 1, 2, 3$) for $\mu_0 = 10^{11}$ GeV where we have assumed $\alpha_3(M_Z) = 0.117$. Also shown is the running of the Yukawa couplings $g_i^2/4\pi$ ($i = t, b, \tau$) with $\tan \beta = 50$. The initial value $g_b^2/4\pi(M_Z)$ has been calculated from $m_b(M_Z) = 3.4$ GeV, where $m_b(M_Z)$ is the $\overline{\text{MS}}$ bottom quark mass at M_Z and we have not included the MSSM superpartner correction (the so-called SUSY correction) to the bottom mass.

⁴The proton is stable in this model [16].

⁵The value quoted by Particle Data Group [48] is: 173.8 ± 5.2 GeV.

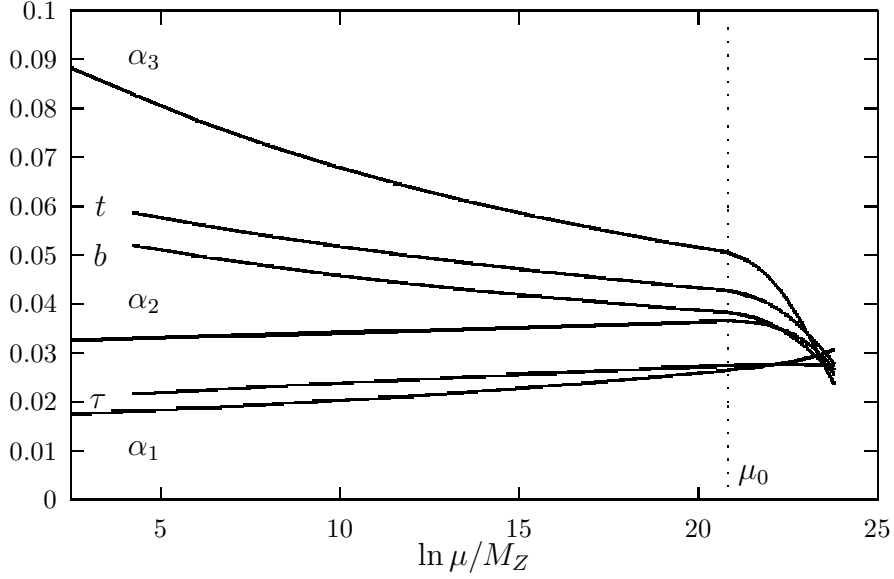


Fig. 1: Running of the gauge and Yukawa couplings for $\tan \beta = 50$ and $\mu_0 = 10^{11}$ GeV.

Now we impose the unification on the gauge couplings with the conventional level of $U(1)_Y$, i.e., $k_Y = 5/3$. Calculating the unification scale M_X and the unified coupling $\alpha(M_X)$ from the input data $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$, we predict $\alpha_3(M_Z)$ as usual. Solid lines in Figs. 2, 3 and 4 show the predicted values of $\alpha_3(M_Z)$, the unification scale M_X and the unified coupling $\alpha(M_X)$, respectively. The predicted value of $\alpha_3(M_Z)$ increases as μ_0 decreases, which has been observed also in ref. [27]. Comparing this with the experimental value [48]

$$\alpha_3(M_Z) = 0.119 \pm 0.002, \quad (66)$$

we see that a lower μ_0 might have a problem with the experimental observation. Of course, the threshold effects at M_X will be very important to give more precise values for $\alpha_3(M_Z)$. But these effects cannot be estimated unless we fix a model above M_X , which is outside of the scope of the present paper.

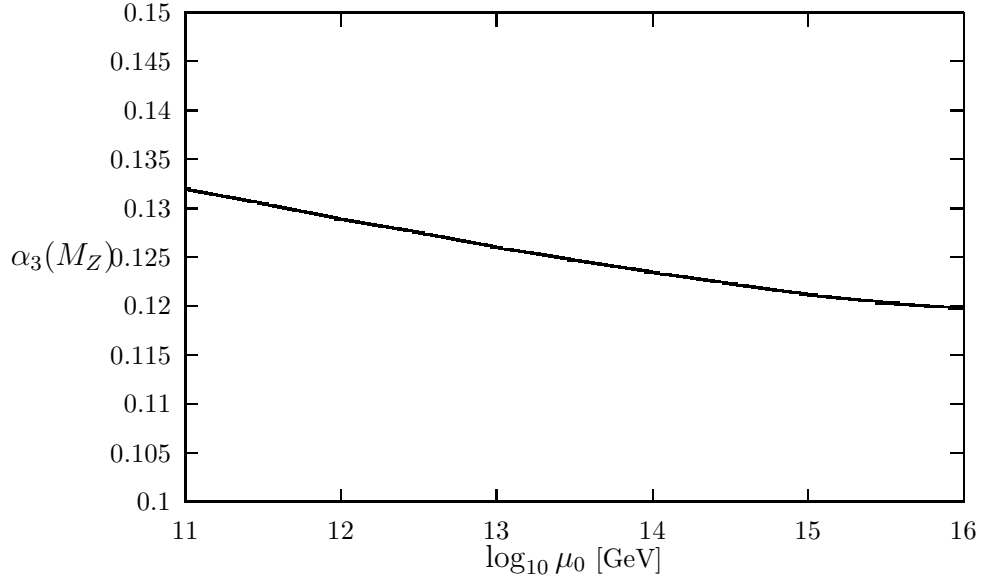


Fig. 2: Prediction of $\alpha_3(M_Z)$

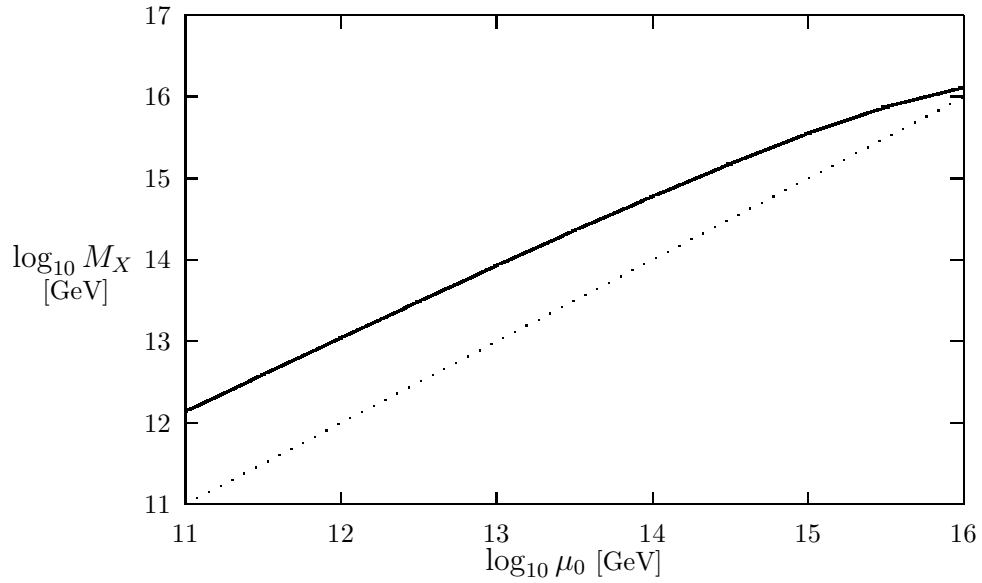


Fig. 3: The unification scale M_X

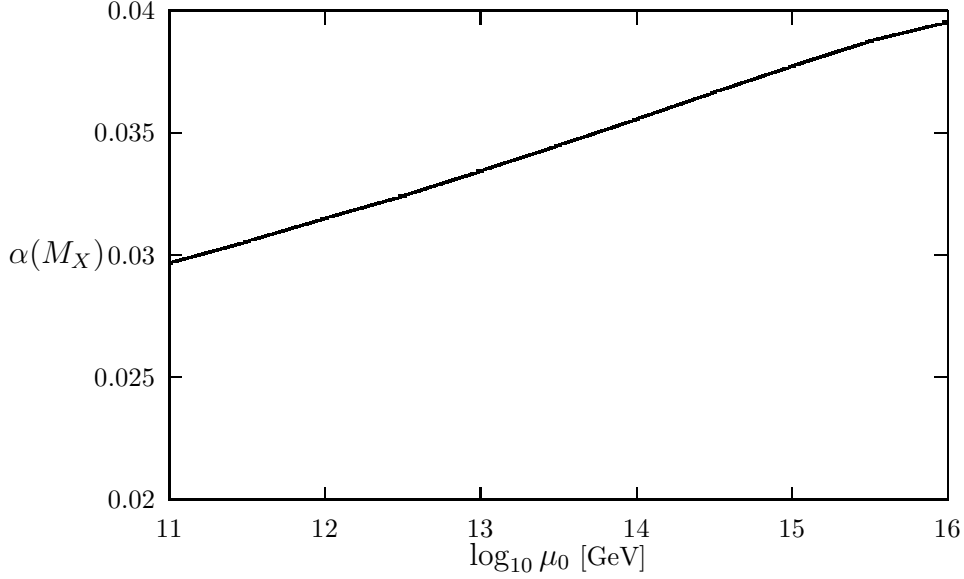


Fig. 4: The unified coupling $\alpha(M_X)$

The level of $U(1)_Y$ denoted by k_Y , which can differ from the conventional value $5/3 \simeq 1.67$ in the framework of string unification [50, 19], could also be responsible for the uncertainty in the prediction of $\alpha_3(M_Z)$. In this case, it is more appropriate to calculate the unification scale M_Y and the unified coupling $\alpha(M_Y)$ from $\alpha_2(M_Z)$ and $\alpha_3(M_Z)$ and then to predict $\alpha_1(M_Z)$. Then from the ratio $\alpha_1(M_Z)/\alpha_Y(M_Z)$, we can obtain the level k_Y , which is shown in Fig. 5, where the dotted line correspond to the conventional level $5/3$. The lower (upper) line in Fig. 5 corresponds to $\alpha_3^{-1}(M_Z) = 8.0$ (9.0). Fig. 6 shows M_Y for the given value of $\alpha_3(M_Z) = 0.117$. To obtain Fig. 5, we have used only the one-loop RG equations, because we are interested in the qualitative change only. As we can see from Fig. 5, different values of $\alpha_3(M_Z)$ do not lead to a large difference in k_Y . It is certainly an interesting approach to regard the level k_Y as a free parameter in performing the analyses that will follow. But we fix k_Y at $5/3$ and use M_X and $\alpha_3(M_Z)$ as well as $\alpha(M_X)$ obtained in Figs. 2, 3 and 4 in the following discussions.

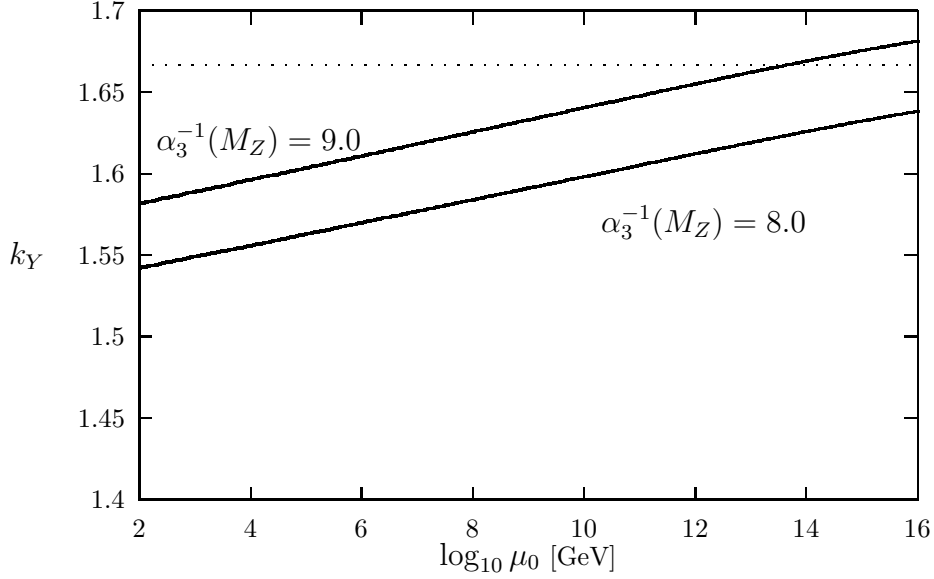


Fig. 5: The level of $U(1)_Y$ k_Y

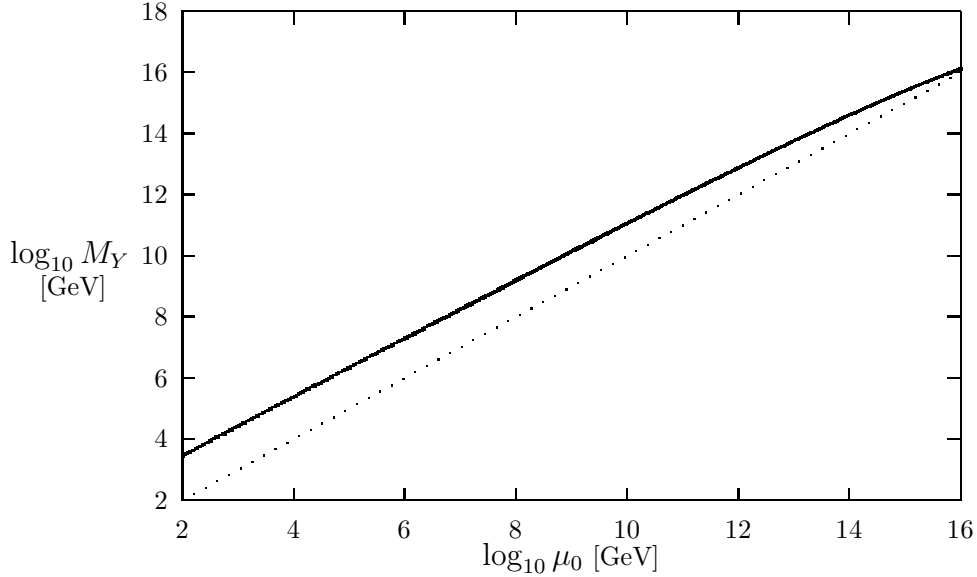


Fig. 6: The unification scale M_Y

3.2 $b - \tau$ unification

The $b - \tau$ Yukawa coupling unification is one of the important aspects in GUTs based on a gauge group like $SU(5)$ or $SO(10)$. Under the assumption of the $b - \tau$ Yukawa unification at M_X , the mass of the bottom quark becomes calculable, and Fig. 7 shows the predicted value of the $\overline{\text{MS}}$ mass $m_b(M_Z)$, where (as before) we have not included the SUSY correction to $m_b(M_Z)$. The upper and lower lines correspond to $\tan \beta = 2$ and 50, respectively. We have

treated $\tan\beta$ as an independent parameter here, although for certain GUTs like a $SO(10)$ GUT it is no longer a free parameter. As we can see from Fig. 7, the predicted value for small $\tan\beta$ increases as μ_0 decreases, while it is relatively stable against the change of μ_0 in most of the region for $\tan\beta = 50$. For example, we have the predicted bottom mass $m_b(M_Z) = 3.4$ GeV for $\mu_0 = 10^{11}$ GeV and $\tan\beta = 50$.

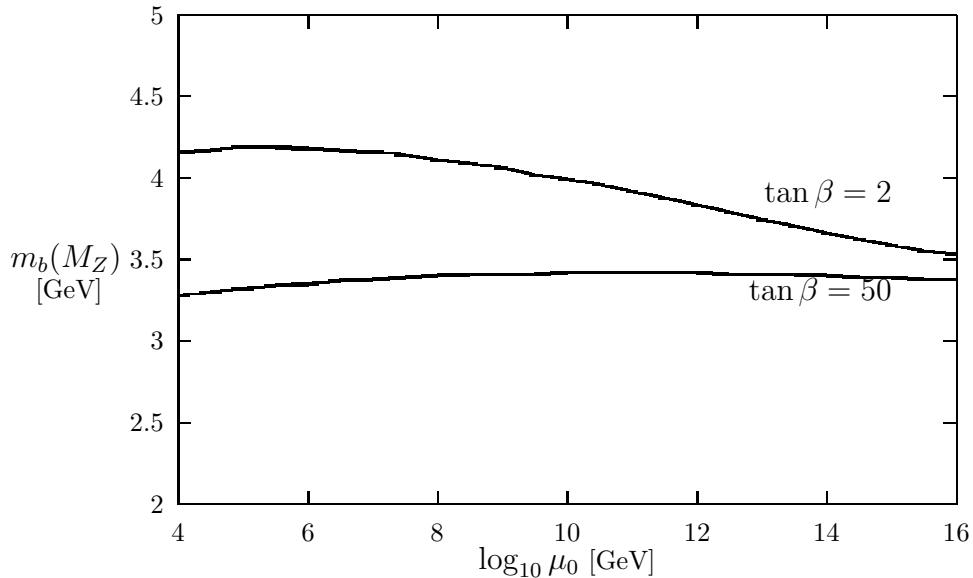


Fig. 7: The bottom mass $m_b(M_Z)$ under the $b - \tau$ Yukawa unification

The present experimental value of the bottom mass contains large uncertainties: Ref. [51], for instance, gives

$$m_b(M_Z) = 2.67 \pm 0.50 \text{ GeV} , \quad (67)$$

while the analysis of the Υ system [52] and the lattice result [53] give $m_b(m_b) = 4.13 \pm 0.06$ GeV and 4.15 ± 0.20 GeV, respectively, ⁶ which translate into

$$m_b(M_Z) = 2.8 \pm 0.2 \text{ GeV}. \quad (68)$$

It is known that the bottom mass can receive a sizable SUSY correction in the large $\tan\beta$ scenario [55]. For $\tan\beta = 50$ it could amount to $O(20 - 30)\%$ [56] and its sign depends on the sign of μ_H . That is, large-radius compactifications prefer large $\tan\beta$. Of course, this SUSY correction depends on the details of the SUSY-mass spectrum, and so we leave the discussion on it to future work.

⁶See also ref. [54].

3.3 SSB sector and radiative breaking of $SU(2) \times U(1)$

Here we use our result on the β -functions on the SSB parameters in the previous and calculate the evolution of the SSB parameters. For simplicity, we consider the universal SSB parameters at M_X , i.e. the universal gaugino masses $M_a(M_X) = M_0$, the universal soft scalar masses $m_i(M_X) = m_0$ and the universal A -parameters $A_i(M_X) \equiv h_i/g_i = A_0$. On top of that, we assume that the relation

$$A_0 = -M_0 \quad (69)$$

is satisfied at the unification scale M_X , which is motivated in the framework of certain superstring theories as well as in the RG invariance consideration [57]-[60].

Solid lines in Fig. 8 show gaugino masses at $M_S = 1$ TeV, where we have taken $M_0=1$ TeV at M_X . All the gaugino masses increase as μ_0 decreases, in accord with the relation (58), i.e., $M_a/\alpha_a = M_0/\alpha(M_X)$, as well as with the μ_0 -dependence of $\alpha(M_X)$ which is shown in Fig. 4. It should be noted that the gaugino masses in Fig. 8 are computed by using the fixed level $k_Y = 5/3$. If we would regard the level as a free parameter and fix it in such a way that all the gauge couplings fit the experimental values at M_Z , we would find a change

$$M_3(M_Z) \simeq 5.3\text{TeV} \rightarrow 4.3\text{TeV}, \quad M_1(M_Z) \simeq 0.83\text{TeV} \rightarrow 0.78\text{TeV} \quad (70)$$

for $\mu_0 = 10$ TeV, for instance, where $M_2(M_Z)$ remains unchanged. So, the net difference compared with the case of the MSSM, is

$$\frac{M_1^{(k_Y)}}{M_1^{(5/3)}} \simeq \frac{k_Y}{5/3} \quad (71)$$

for a fixed value of $M_2/\alpha_2(M_Z) = M_3/\alpha_3(M_Z)$, where $1.0 \gtrsim k_Y/(5/3) \gtrsim 0.9$ as we can see from Fig. 5.

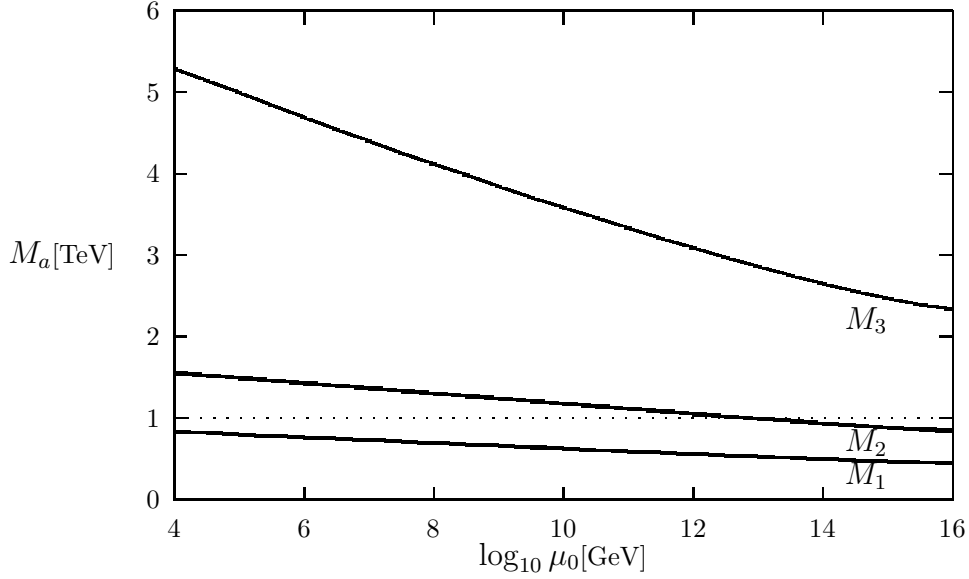


Fig. 8: Gaugino masses

Solid lines in Fig.9 show sfermion masses at the SUSY scale M_S . In this figure, \tilde{Q} , \tilde{L} and \tilde{E} stand for sfermions of the quark doublet, the lepton doublet and the lepton singlet for the first two families, for which we have neglected the contribution of the Yukawa couplings to their evolution. We have taken $\tan\beta = 2$, $M_0 = 1$ TeV and $m_0 = 0.8$ TeV. We expect that the μ_0 -dependence of the squarks masses is large, because the gaugino masses dominantly contribute to the evolution of the squark masses of the first two generations. In fact, the μ_0 -dependence of the gaugino masses shown in Fig. 8 is reflected in that as μ_0 decreases, the squarks become much heavier than the sleptons, as we can see in Fig.9 . Similarly, we can calculate the sfermion masses of the third family. These masses will be shown later after the discussion of radiative electroweak symmetry breaking.

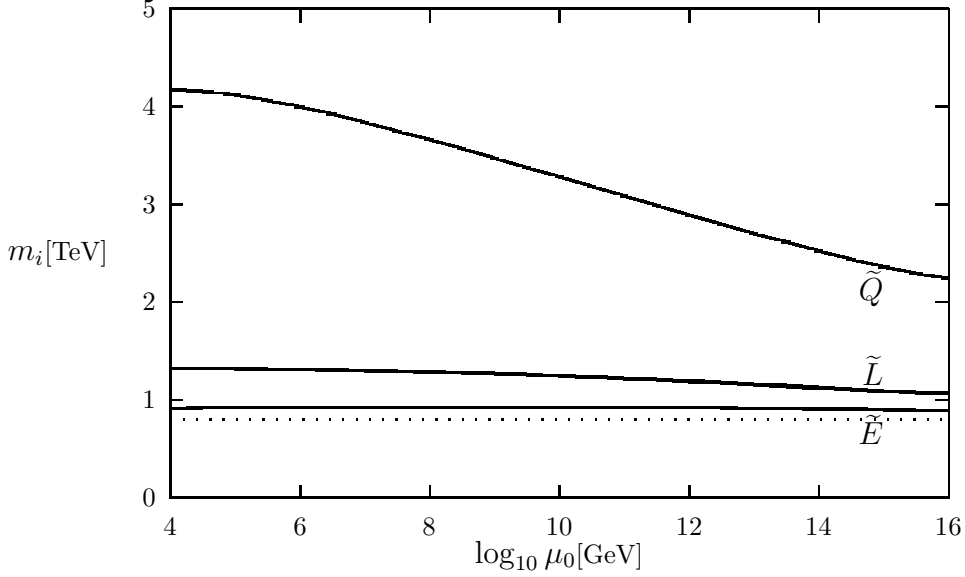


Fig.9: Sfermion masses

Now we come to discuss the radiative electroweak symmetry breaking. We fix the values of μ_H and B by using the two minimization conditions of the Higgs potential at the weak scale,

$$m_1^2 + m_2^2 = -\frac{2\mu_H B}{\sin 2\beta}, \quad (72)$$

$$m_1^2 - m_2^2 = -\cos 2\beta (M_Z^2 + m_1^2 + m_2^2), \quad (73)$$

where $m_{1,2}^2 = m_{H_d, H_u}^2 + \mu_H^2$. For the desired electroweak symmetry breaking to occur, the condition

$$m_1^2 m_2^2 < |\mu_H B|^2 \quad (74)$$

should be satisfied, while the bounded-from-below condition along the D -flat direction in the Higgs potential requires

$$m_1^2 + m_2^2 > 2 |\mu_H B|. \quad (75)$$

As we have noticed at the end of the previous section, the β -functions for μ_H , B , $m_{H_u}^2$ and $m_{H_d}^2$ below μ_0 do not change when passing the Kaluza-Klein threshold μ_0 . We therefore expect that the existence of a large compactification radius R in the present model has, through the other parameters whose β -functions change at the threshold, only an indirect

influence on the evolution of these parameters. We have in fact found that there exists a wide region in the parameter space (M_0, m_0) leading to the desired electroweak breaking. For example, the electroweak symmetry breaking always occurs for the region, $m_0 \leq O(M_0)$, $\mu_0 = 10^5 \sim 10^{16}$ GeV and $\tan \beta = 2 \sim 50$.

We have found that the stau (mass)² becomes easily negative in the large $\tan \beta$ scenario. Similarly, the stau mass becomes easily smaller than the lightest neutralino mass, in particular in the large $\tan \beta$ scenario. In such a case the LSP is the stau which is electrically charged and should have been observed if the R parity is not violated⁷. Therefore, it is important to compare the masses of the lightest neutralino and the stau. Figs. 10 and 11 show the lightest stop \tilde{t} , sbottom \tilde{b} , stau $\tilde{\tau}$ and neutralino χ^0 masses for $\tan \beta = 2$ and 50, respectively. We have taken $M_0 = 1$ TeV, $m_0 = 0.8$ TeV and $m_b(M_Z) = 2.7$ GeV without assuming the $b - \tau$ Yukawa unification. In the case considered above the stau becomes the LSP below $\mu_0 = 10^{13.5}$ GeV if $\tan \beta = 50$ (See Fig. 11).

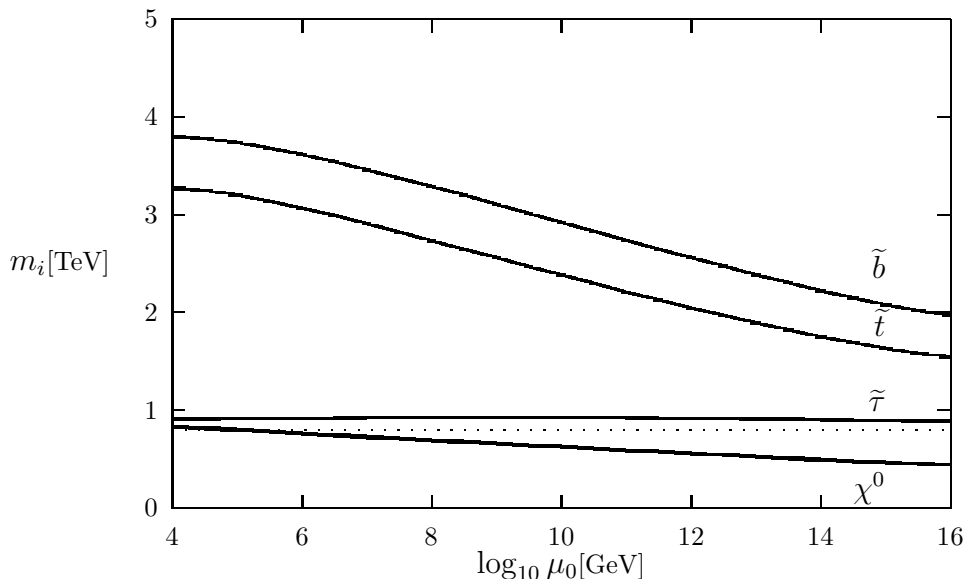


Fig. 10: S-spectrum for $\tan \beta = 2$

⁷ In refs.[60, 61] the experimental constraints have been considered in details for (finite) $SU(5)$ GUTs and $SO(10)$ GUTs.

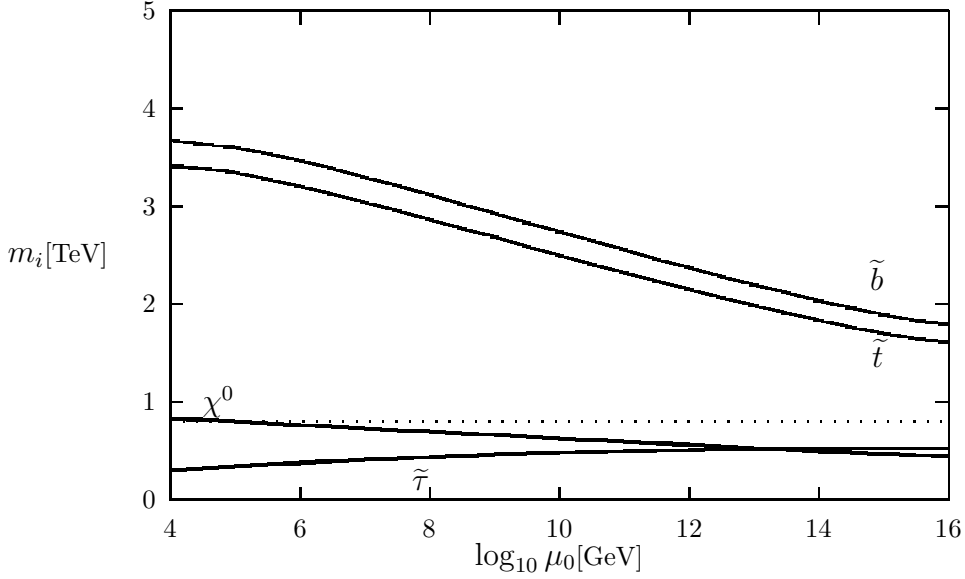


Fig. 11: S-spectrum for $\tan \beta = 50$

Figs. 12, 13 and 14 show the $m_{\tilde{\tau}}^2 > 0$ constraint for $\mu_0 = 10^{16}$, 10^{11} and 10^5 GeV, where we vary $\tan \beta$ from $\tan \beta = 2$ to 50. The parameter range in the (M_0, m_0) space shown in these figures always leads to a successful electroweak symmetry breaking. The asterisks denote the region leading to a negative stau (mass)², and the open squares stand for the region where the stau is lighter than the lightest neutralino. These figures show a similar feature of the allowed range in the $(m_0, \tan \beta)$ space for a wide range of μ_0 .

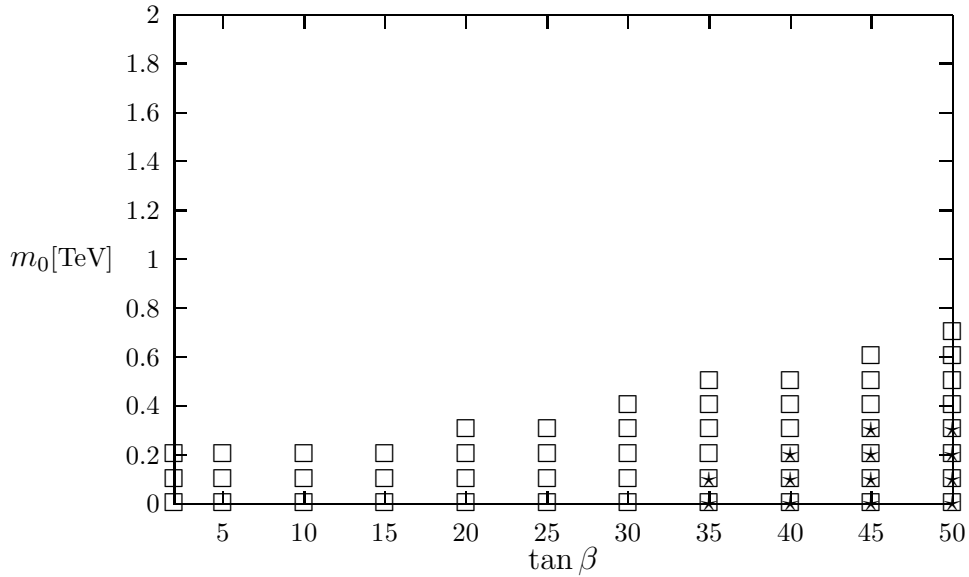


Fig. 12: The stau mass and the LSP for $\mu_0 = 10^{16}$ GeV

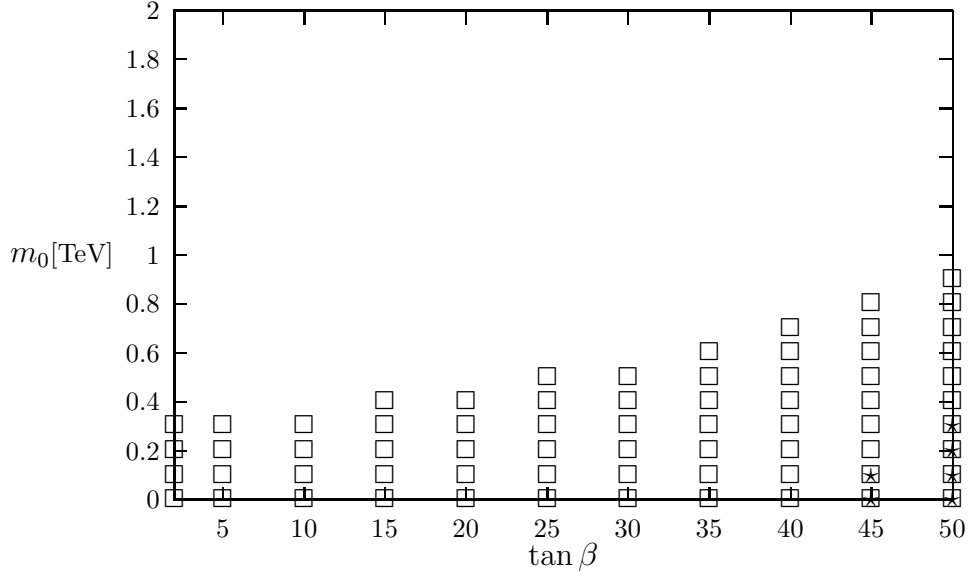


Fig. 13: The stau mass and the LSP for $\mu_0 = 10^{11}$ GeV

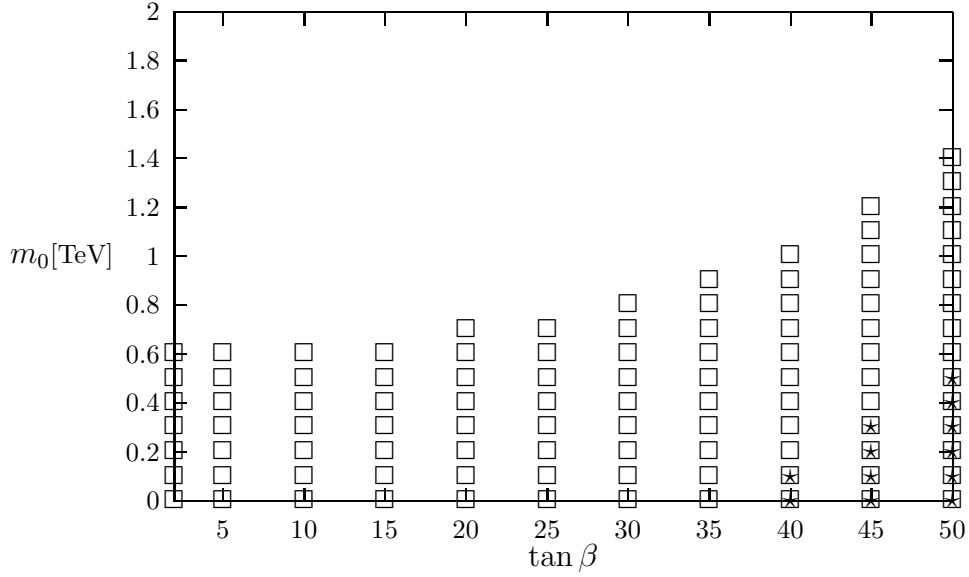


Fig. 14: The stau mass and the LSP for $\mu_0 = 10^5$ GeV

4 Conclusion

If one extrapolates the MSSM to higher dimensions so that the compactification scale is significantly smaller than the ordinary SUSY-GUT scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, the massive Kaluza-Klein states modify the qualitative nature of the ordinary unification scenario [3]-[32]. In this paper we have illustrated how to apply the non-perturbative RG technique [33]-[35]

to handle the problems associated with large-radius compactifications. As an application we have derived the matching condition between the effective, renormalizable and original, non-renormalizable theories from the requirement that the β -functions calculated in the effective theory go over in the large-compactification-radius limit to those calculated using the non-perturbative RG technique with the assumption that the space-time dimensions are not compactified. We have not followed up this powerful RG technique further, but we would like to mention that the presence of higher-dimension operators, for instance, can be easily taken into account in this method.

Given the matching condition, we have decided to stay in the renormalizable dimension, as Dienes *et al.* [16], because if we ignore (provably higher order) effects such as those coming from higher-dimension operators, their method is simple and convenient to calculate the contributions of the massive Kaluza-Klein states to the RG evolution of couplings. Furthermore, as long as we want to stay in the one-loop approximation, we can simply extend the treatment, due to the recent development on renormalization of the SSB parameters [36]-[39], to include the SSB sector: We have computed the one-loop β -functions of the SSB parameters in the MSSM which is extrapolated to $D = 4 + \delta$ dimensions under the assumption that the Kaluza-Klein towers exist only in the gauge supermultiplets and Higgs supermultiplets [16].

We have addressed ourselves to various phenomenological issues in the model. We first have confirmed the observation of Ghilencea and Ross that the value of $\alpha_3(M_Z)$ predicted from the gauge coupling unification increases as μ_0 decreases, and could easily exceed the experimental value 0.119 ± 0.002 if no other corrections such as the threshold corrections at M_{GUT} are taken into account. In contrast to this, the $b - \tau$ unification can be obtained in a relatively wide range of μ_0 for a large $\tan\beta$, and we have found that large-radius compactifications prefer large $\tan\beta$. Another finding is that the relation among the ratios M_i/g_i^2 (58) holds in one-loop order in the model so that if one takes account the level of $U(1)_Y$ appropriately, the model with small μ_0 differs from the conventional MSSM in this sector of the SSB parameters. We have also found that the μ_0 -dependence of the sfermion masses, especially those of the squarks, is quite large.

From our analyses in this paper, we would like to conclude that there exist a certain chance to experimentally discriminate the model from the MSSM even if the massive Kaluza-

Klein states are so heavy that they are not accessible in future collider experiments.

Acknowledgment

Two of us (J.K and G.Z) would like to thank the Theory Group of the Max-Planck-Institute for Physics, Munich and the Humboldt University, Berlin for their hospitality, where part of this work has been done. J.K would like to thank H. Terao for useful discussions.

References

- [1] E. Witten, Nucl. Phys. **B471** (1996) 135; P. Horava and E. Witten, Nucl. Phys. **B475** (1996) 94.
- [2] E. Witten, Nucl. Phys. **B443** (1995) 85; A. Dabholkar, Phys. Lett. **B357** (1995) 307; C.M. Hull, Phys. Lett. **B357** (1995) 545; J. Polchinski and E. Witten, Nucl. Phys. **B460** (1996) 525.
- [3] I. Antoniadis and M. Quiros, Phys. Lett. **B392** (1997) 61.
- [4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “*New Dimensions at a Millimeter to a Fermi and Superstring at TeV*”, hep-ph/9804398.
- [5] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “*The Hierarchy Problem and New Dimensions at a Millimeter*”, hep-ph/9803315; “*Phenomenology, Astrophysics and Cosmology of Theories with Sub-Millimeter Dimensions and TeV Scale Quantum Gravity*”, hep-ph/9807344.
- [6] N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, “*Stabilization of Sub-Millimeter Dimensions: The New Guise of the Hierarchy Problem*”, hep-th/9809124.
- [7] I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quirós, “*Soft Masses in Theories with Supersymmetry Breaking by TeV-Compactification*”, hep-ph/9810410.
- [8] N. Arkani-Hamed and S. Dimopoulos, “*New origin for approximate symmetries from distant breaking in extra dimensions*”, hep-ph/9811353.
- [9] Z. Berezhiani and G. Dvali, “*Flavor Violation in Theories with TeV Scale Quantum Gravity*”, hep-ph/9811378.
- [10] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russell, “*Neutrino Masses from Large Extra Dimensions*”, hep-ph/9811448.
- [11] R. Sundrum, “*Effective Field Theory for a Three-Brane Universe*”, hep-ph/9805471; “*Compactification for a Three-Brane Universe*”, hep-ph/9807348.

- [12] G. Shiu and S.-H.H. Tye, “*TeV Scale Superstring and Extra Dimensions*”, hep-th/9805157.
- [13] A. Pomarol and M. Quirós, “*The Standard Model from extra dimensions*”, hep-ph/9806263.
- [14] J. Lykken, E. Poppitz and S. Trivedi, “*Branes with GUTs and Supersymmetry Breaking*”, hep-th/9806080.
- [15] E. Poppitz, “*Chirality, D-Branes and Gauge/String Unification*”, hep-ph/9806309.
- [16] K. Dienes, E. Dudas and T. Gherghetta, “*Grand Unification at Intermediate Mass Scales through Extra Dimensions*”, hep-ph/9806292.
- [17] C. Bachas, “*Unification with Low String Scale*”, hep-ph/9807415.
- [18] Z. Kakushadze and S.-H. Tye, “*Three Generation in Type I Compactifications*”, hep-th/9806143. “*Brane World*”, hep-th/9809147.
- [19] K. Benakli, “*Phenomenology of Low Quantum Gravity Scale Models*”, hep-ph/9809582.
- [20] K. Dienes, E. Dudas, T. Gherghetta and A. Riotto, “*Cosmological Phase Transitions and Radius Stabilization in Higher Dimensions*”, hep-ph/9809406.
- [21] L. Randall and R. Sundrum, “*Out of This World Supersymmetry Breaking*”, hep-th/9810155.
- [22] C.P. Burgess, L.E. Ibáñez and F. Quevedo, “*Strings at the Intermediate Scale or is the Fermi Scale Dual to the Planck Scale ?*”, hep-ph/9810535.
- [23] T. Han, J.D. Lykken and R.J. Zhang, “*On Kaluza-Klein States from Large Extra Dimensions*”, hep-ph/9811350.
- [24] K.R. Dienes, E. Dudas and T. Gherghetta, “*Neutrino Oscillations without Neutrino Masses or Heavy Mass Scales: A Higher-Dimensional Seesaw Mechanism*”, hep-ph/9811428.

- [25] Z. Kakushadze, “*Novel Extension of MSSM and ”TeV Scale” Coupling Unification*”, hep-th/9811193.
- [26] S. Abel and S. King, “*On fixed points and fermion mass structure from large extra dimensions*”, hep-ph/9809467.
- [27] D. Ghilencea and G.G. Ross, “*Unification and extra space-time dimensions*”, hep-ph/9809217.
- [28] H. Hatanaka, T. Inami and C.S. Lim, “*The Gauge Hierarchy Problem and Higher Dimensional Gauge Theories*”, hep-th/9805067.
- [29] G.F. Giudice, R. Rattazzi and J.D. Wells, “*Quantum Gravity and Extra Dimensions at High-Energy Colliders*”, hep-ph/9811291.
- [30] S. Nussinov and R. Shrock, “*Some Remarks on Theories with Large Compact Dimensions and TeV-Scale Quantum Gravity*”, hep-ph/9811323.
- [31] E.A. Mirabelli, M. Perelstein and M.E. Peskin, “*Collider Signatures of New Large Space Dimensions*”, hep-ph/9811337.
- [32] J. L. Hewett, “*Indirect Collider Signals for Extra Dimensions*”, hep-ph/9811356.
- [33] K. Wilson and J. Kogut, Phys. Rep. **12C** (1974) 75.
- [34] F.J. Wegner and A. Houghton, Phys. Rev. **A8** (1973) 401.
- [35] J. Polchinski, Nucl. Phys. **B231** (1984) 269; C. Wetterich, Phys. Lett. **B301** (1993) 90; N. Teradis and C. Wetterich, Nucl. Phys. **B442** [FS] (1994) 541.
- [36] Y. Yamada, Phys.Rev. **D50** (1994) 3537.
- [37] J. Hisano and M. Shifman, Phys. Rev. **D56** (1997) 5475.
- [38] I. Jack and D.R.T. Jones, Phys. Lett. **B415** (1997) 383; I. Jack, D.R.T. Jones and A. Pickering, Phys. Lett. **B426** (1998) 73.
- [39] L.V. Avdeev, D.I. Kazakov and I.N. Kondrashuk, Nucl. Phys. **B510** (1998) 289.

- [40] T.R. Morris, Int. J. Mod. Phys. **A9** (1994) 2411.
- [41] J.F. Nicoll, T.S. Chang and H.E. Stanley, Phys. Rev. **A13** (1976) 1251.
- [42] A. Hasenfratz and P. Hasenfratz, Nucl. Phys. **B270** (1986) 687.
- [43] T.R. Morris, Phys. Lett. **334** (1994) 355; K.-I. Aoki *et al*, Prog. Theor. Phys. **95** (1996) 409; T.R. Morris, *Elements of the Continuous Renormalization Group*, hep-th/9802039 and references therein.
- [44] I. Jack, D.R.T. Jones, S.P. Martin, M.T. Vaughn and Y. Yamada, Phys. Rev. **D50** (1994) R5481.
- [45] T. Kobayashi, J. Kubo and G. Zoupanos, Phys. Lett. **B427** (1998) 291.
- [46] I. Jack, D.R.T. Jones and A. Pickering, Phys. Lett. **B432** (1998) 114.
- [47] V. Novikov, M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. **B229** (1983) 381; Phys. Lett. **B166** (1986) 329; M. Shifman, Int. J. Mod. Phys. **A11** (1996) 5761 and references therein.
- [48] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. **C3** (1998) 1.
- [49] J. Kubo, M. Mondragón, M. Olechowski and G. Zoupanos, Nucl. Phys. **B479** (1996) 25.
- [50] L.E. Ibáñez, Phys. Lett. **B318** (1993) 73; H. Kawabe, T. Kobayashi and N. Ohtsubo, Phys. Lett. **B325** (1994) 77; Nucl. Phys. **B434** (1995) 210.
- [51] S. Martí i García, J. Fuster and S. Cabrera, hep-ex/9708030.
- [52] M. Jamin and A. Pich, Nucl. Phys. **B507** (1997) 334.
- [53] V. Giménez, G. Martinelli and C.T. Sachrajda, Phys. Lett. **B393** (1997) 124.
- [54] G. Rodrigo, A. Santamaria and M. Bilenky, Phys. Rev. Lett. **79** (1997) 193.
- [55] L.J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. **D50** (1994) 7048; R. Hempfling, Phys. Rev. **D49** (1994) 6168.

- [56] M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. **B426** (1994) 269.
- [57] A. Brignole, L.E. Ibáñez and C. Muñoz, Nucl. Phys. **B422** (1994) 125; A. Brignole, L.E. Ibáñez, C. Muñoz and C. Scheich, Z. Phys. **C74** (1997) 157.
- [58] D.R.T. Jones, L. Mezincescu and Y.P. Yau, Phys. Lett. **B148** (1984) 317; I. Jack and D.R.T. Jones, Phys. Lett. **B349** (1995) 294; D.I. Kazakov, M.Yu. Kalmykov, I.N. Kondrashuk and A.V. Gladyshev, Nucl. Phys. **B471** (1996) 387.
- [59] Y. Kawamura, T. Kobayashi and J. Kubo, Phys. Lett. **B405** (1997) 64.
- [60] T. Kobayashi, J. Kubo, M. Mondragón and G. Zoupanos, Nucl. Phys. **B511** (1998) 45.
- [61] Y. Kawamura, T. Kobayashi and H. Shimabukuro, Phys. Lett. **B436** (1998) 108.